Boletín de la SOCIEDAD MATEMATICA MEXICANA

Tercera Serie
Volumen 1
Número 2
1995

Contenido

ARTÍCULOS DE INVESTIGACIÓN

SOCIEDAD MATEMATICA MEXICANA

Bol. Soc. Mat. Mexicana (3) vol. 1, 1995

NEW CONSTRUCTIONS FOR INTEGRAL SUMS OF SQUARES FORMULAE

BY DAVID ROMERO

For given positive integers r and s, consider the problem of determining the smallest integer n for which there exists a sums of squares formula of the form

$$(x_1^2 + \cdots + x_r^2)(y_1^2 + \cdots + y_s^2) = (z_1^2 + \cdots + z_n^2), \tag{1}$$

in which $z_1, \ldots, z_n$ are bilinear forms in $x_1, \ldots, x_r, y_1, \ldots, y_s$ with integer coefficients. The concept of signed intercalate matrices allowed Smith and Yiu [SY] to establish some of the best known upper bounds for n, whenever $10 \leq r \leq s$ and $17 \leq s \leq 32$. By means of new elementary constructions we improve here on six of these bounds.

Since the pioneering paper of Hurwitz in 1898 [H], a huge amount of work (excellent accounts are given in [SH1] and [SH2]) has been devoted to the difficult problem of determining the values r, s, and n for which there exist formulas of the type (1), where $X = (x_1, \ldots, x_r)$ and $Y = (y_1, \ldots, y_s)$ are systems of indeterminates, and each z_k $(k = 1, \ldots, n)$ is bilinear in X and Y with coefficients in a commutative ring $\mathscr{R}$. Whenever we can construct this formula, we say that the triple of integers (r, s, n) is *admissible* over $\mathscr{R}$. It is well known ([YUZ], [Y1]) the equivalence of the problem of finding admissible triples over the integers, with that of constructing consistently signed intercalate matrices. This combinatorial approach led to several new admissible triples, and is proving to be a formidable tool in this sense (see [Y2] and [SY]). In the same vein, but from constructions that cannot be obtained by permuting dyadic intercalate matrices, we display here additional admissible triples over the integers.

The two following definitions are from [SY]. Let M be an $r \times s$ matrix with generic entry $M(i, j)$. We shall think of the entries of M as *colors*.

Definition (1). An *intercalate matrix* of type (r, s, n) is an $r \times s$ matrix M with n distinct colors and satisfying:

(i) The colors along each row or column are distinct.
(ii) If $M(i, j) = M(k, \ell)$, then $M(i, \ell) = M(k, j)$.

Definition (2). An intercalate matrix M can be *signed consistently* if it is possible to endow each entry $M(i, j)$ with a sign $\epsilon_{ij} = \pm 1$ such that $\epsilon_{ij}\epsilon_{i\ell}\epsilon_{kj}\epsilon_{k\ell} = -1$ whenever $M(i, j) = M(k, \ell)$, for $i \neq k$, $j \neq \ell$.

Bol. Soc. Mat. Mexicana (3) vol. 1, 1995

THE GEOMETRY AND COHOMOLOGY OF M_{12}: II

BY R. JAMES MILGRAM† AND MICHISHIGE TEZUKA

In this note we complete the discussion of the cohomology of the sporadic group M_{12} which was initiated in [AAM]. There we determined $H^*(M_{12};\mathbb{F}_2)$. Here we determine $H^*(M_{12};\mathbb{F}_3)$. Our main result is

THEOREM (A). *$H^*(M_{12};\mathbb{F}_3)$ is Cohen-Macaulay (i.e. free and finitely generated) over a polynomial algebra $\mathbb{F}_3[Q, L^2]$ where Q has dimension 12 and L^2 has dimension 16. $H^*(M_{12};\mathbb{F}_3)$ has Poincaré series*

$$\frac{\mathscr{Q}}{(1-x^{12})(1-x^{16})}$$

where $\mathscr{Q}$ is

$$1+x^3+2x^4+x^5+x^7+2x^8+2x^9+3x^{10}+4x^{11}+2x^{12}+2x^{13}+2x^{14}+4x^{15} + 3x^{16}+2x^{17}+2x^{18}+x^{19}+x^{21}+2x^{22}+x^{23}+x^{26}.$$

In fact we give explicit generators and determine the ring structure (5.5–5.9).

REMARK: $H^*(GL_3(3);\mathbb{F}_3)$ has the same Sylow 3-subgroup as M_{12} and the determination of generators for $H^*(GL_3(3);\mathbb{F}_3)$ in [TY] shows that the two restriction maps have the same image in $H^*(Syl_3(M_{12});\mathbb{F}_3)$. Consequently these two rings are isomorphic.

It is useful to know $H^*(M_{12};\mathbb{F}_3)/\mathscr{P}_i$ where the $\mathscr{P}_i$ run over the minimal prime ideals in $H^*(M_{12};\mathbb{F}_3)$. In particular these integral domains are important in studying the modular representations of M_{12}. We have

COROLLARY (B). *There are three minimal prime ideals $\mathscr{P}_I$, $\mathscr{P}_{II}$, and $\mathscr{P}_{III}$ in $H^*(M_{12};\mathbb{F}_3)$, and the quotients are given as*

$$H^*(M_{12};\mathbb{F}_3)/\mathscr{P}_I \cong H^*(M_{12};\mathbb{F}_3)/\mathscr{P}_{II}$$

while $H^*(M_{12};\mathbb{F}_3)/\mathscr{P}_{III} \cong \mathbb{F}_3[Q,t^2]$ where t^2 has dimension 4.

†Research partially supported by a grant from the N.S.F.

Bol. Soc. Mat. Mexicana (3) vol. 1, 1995

SPECIAL UNIFORMITIES

BY ADALBERTO GARCÍA-MÁYNEZ

When a uniform space $(X, \mathscr{U})$ is completed, the completion $\widehat{X}$ is not necessarily embedable in the Stone-Čech compactification βX of X. However, if $(X, \mathscr{U})$ is a BU space, $\widehat{X}$ is homeomorphic to the subspace of βX consisting of all z-ultrafilters whose corresponding superset filter is Cauchy in $(X, \mathscr{U})$. (See [4;7.43]). If every finite cozero cover of X belongs to $\mathscr{U}$, the uniform space $(X, \mathscr{U})$ is necessarily BU. In this paper we consider some classes of BU uniform spaces and study their completions in βX. Restricting ourselves to a simpler kind of cozero covers, which include finite ones (the so called *special covers*), we get uniformity basis which may not be equivalent to the traditional ones but yield the same completions in a more natural way. In a future paper, we plan to use special covers in the study of the Samuel compactification of $(X, \mathscr{U})$.

All spaces we consider in this paper are completely regular and Hausdorff and all maps are continuous.

Definition (1). A uniform space $(X, \mathscr{U})$ is BU if every map $f: (X, \mathscr{U}) \to [0, 1]$ is uniformly continuous.

THEOREM (1). See [3, Theorem 6]. *The uniform space $(X, \mathscr{U})$ is BU if and only if the identity map $j: (X, \mathscr{U}) \to (X, \mathscr{U}_0)$ is uniformly continuous, where $\mathscr{U}_0$ denotes the collection of finite cozero covers of X.*

Definition (2). A cover α of a space X is *special* if α is a finite union $\alpha = \alpha_1 \cup \alpha_2 \cup \cdots \cup \alpha_m$ of discrete cozero families in X.

Note that every special cover is locally finite and σ-discrete.

Definition (3). A uniform space $(X, \mathscr{U})$ is *special* if $\mathscr{U}$ has a basis consisting of special covers.

Definition (4). A cover α of a space X is *star finite* (respectively, *star countable*) if for every $A \in \alpha$ the collection $\{B \in \alpha | B \cap A \neq \emptyset\}$ is finite (respectively, countable).

Clearly, every star finite open cover of a space X is locally finite.

Definition (5). A cover α of a space X is *star bounded* if there exists a natural number m such that for each $A \in \alpha$, the collection $\{B \in \alpha | B \cap A \neq \emptyset\}$ has at most m elements.

Bol. Soc. Mat. Mexicana (3) vol. 1, 1995

CONTINUITY OF SMALL EIGENFUNCTIONS ON DEGENERATING RIEMANN SURFACES WITH HYPERBOLIC CUSPS

BY JONATHAN HUNTLEY, JAY JORGENSON AND ROLF LUNDELIUS

Introduction

Let M denote a hyperbolic Riemann surface of finite volume. If M is compact, then the spectrum of the Laplacian which acts on the space of smooth functions on M is purely discrete, whereas if M is non-compact then both discrete and continuous spectrum exists, and the continuous spectrum is represented by the band $[1/4, \infty)$ with multiplicity and measure determined by the surface. In other words, the spectrum of the Laplacian below the eigenvalue $1/4$ is discrete in both the compact and non-compact settings (see [Se]). A fundamental question one can pose is to understand the spectrum of the Laplacian on a degenerating family of hyperbolic Riemann surfaces when the limit surface lies in the Deligne-Mumford (stable) boundary of the moduli space of (possibly noded) algebraic curves of fixed signature. In this note, we shall use the heat kernel convergence theorem as proved in [JL3] to prove convergence of the small eigenfunctions, meaning eigenfunctions whose corresponding eigenvalue is less than $1/4$. Our results go beyond that of other authors, such as [CC], [He], and [Ji] in that we consider non-compact as well as compact family of hyperbolic surfaces, and, more importantly, our technique of proof immediately extends to a larger class of degeneration problems. In particular, we shall discuss how to prove a heat kernel convergence theorem and, consequently, convergence of the small eigenfunctions in the setting of degenerating Riemann surfaces with finite volume metrics which are fixed, compactly supported conformal perturbations of the hyperbolic metric. Finally, let us note here that in [DJ] the authors follow a method of proof similar to that of this article to establish continuity of small eigenfunctions on a sequence of degenerating family of hyperbolic three manifolds of finite volume.

The first author acknowledges support from NSF grant DMS-92-03393, from MSRI under NSF grant DMS-90-22140, and from several PSC-CUNY grants. The second author acknowledges support from NSF grant DMS-93-07023, from the Max-Planck-Institut für Mathematik, and from the Sloan Foundation. Part of the research of the third author was conducted during the US-Sweden Workshop on Spectral Methods sponsored by the NSF under grant INT-9217529.

Bol. Soc. Mat. Mexicana (3) vol. 1, 1995

ON THE CONTINUITY OF WIENER CHAOS

BY YAOZHONG HU AND VÍCTOR PÉREZ-ABREU

Abstract

The paper studies the problem of when a chaos expansion in terms of multiple Wiener-Itô integrals $\sum_{m=0}^{\infty} I_m(f_m)$ possesses a Lipschitz continuous extension. It is shown that in the finite sum case, a necessary and sufficient condition is that, for each $m \geq 1$, $I_m(f_m)$ has a continuous extension. In the infinite sum situation a sufficient condition is given.

1. Introduction

Let $C_0 = C_0(T)$ be the space of real valued continuous functions on $T = [0, 1]$ which are zero at zero, and let $(\Omega, \mathscr{F}, P) = (C_0, \mathscr{B}(C_0), \mu)$ be the *Wiener space*, that is, μ is the standard Wiener probability measure on the Borel σ-field $\mathscr{B}(C_0)$. Let $W = \{W_t; t \in T\}$ be the Wiener process defined on $(\Omega, \mathscr{F}, P)$, let $\mathscr{F}^W$ be the σ-field generated by W and let $L^2(W) = L^2(\Omega, \mathscr{F}^W, P)$ be the *Wiener chaos*. A function in the space $L^2(W)$ is called a *nonlinear Wiener functional*.

In a basic paper in 1951, K. Itô ([4]) proved that any $F \in L^2(W)$ admits the L^2-orthogonal expansion

$$F = \sum_{m=0}^{\infty} I_m(f_m), \tag{1.1}$$

where for each $m \geq 1$, $I_m(f_m)$ is the *m-th multiple Wiener-Itô integral* of the non-random *kernel* $f_m \in L^2(T^m) = L^2(T^m, \mathscr{B}(T^m), \lambda^m)$, $I_0(f_0) = f_0 = \int_\Omega F(\omega)\, dP(\omega)$ and

$$\sum_{m=0}^{\infty} m!\, \|f_m\|^2_{L^2(T^m)} < \infty. \tag{1.2}$$

The Multiple Wiener-Itô integral $I_m(f_m)$ is defined as the iterated Itô stochastic integral

$$I_m(f_m) = m! \int_0^1 \left\{ \int_0^{t_{m-1}} \cdots \left\{ \int_0^{t_2} \widetilde{f}_m(t_1, \ldots, t_m) dW_{t_1} \right\} \cdots dW_{t_{m-1}} \right\} dW_{t_m}, \tag{1.3}$$

Bol. Soc. Mat. Mexicana (3) Vol. 1, 1995

ON THE CHARACTERIZATION OF Q-SUPERLINEAR CONVERGENCE OF QUASI-NEWTON INTERIOR-POINT METHODS FOR NONLINEAR PROGRAMMING

BY H. J. MARTINEZ, Z. PARADA AND R. A. TAPIA

Abstract

In this paper we extend the well-known Boggs-Tolle-Wang characterization of Q-superlinear convergence for quasi-Newton methods for equality constrained optimization to quasi-Newton interior-point methods for nonlinear programming. Critical issues in this extension include, the choice of the centering parameter, the choice of the steplength parameter, and the determination of the primary variables.

1. Introduction

In 1974 Dennis and Moré [3] gave a characterization of those quasi-Newton methods for the nonlinear equation problem which produce iterates which are Q-superlinearly convergent. This characterization immediately carries over to unconstrained optimization by working with the nonlinear equation (gradient equal to zero) that results from the first-order necessary conditions. Similarly the Dennis-Moré characterization can be carried over to equality constrained optimization by working with the nonlinear system corresponding to the first-order necessary conditions. This nonlinear system, involves the two groups of variables (x, y). Here x is the vector of primal variables, and y is the vector of dual variables corresponding to the equality constraints. Hence the approach characterizes Q-superlinear convergence in terms of the variable-pair (x, y). Indeed, the first authors to establish Q-superlinear convergence for various secant methods for equality constrained optimization, Han [8] in 1976, Tapia [12] in 1977, and Glad [7] in 1979, did so using this approach and established Q-superlinear convergence in the pair (x, y). Not long after, in 1982, Boggs, Tolle, and Wang [1] observed that under certain assumptions,

1991 *Mathematics Subject Classification*. 65K, 49M, 90C.

Key words and phrases: Interior-point methods, primal-dual variables, nonlinear programming, superlinear convergence, Boggs-Tolle-Wang characterization, Dennis-Moré characterization.

Research sponsored by NSF Cooperative Agreement No. CCR-88-09615, ARO Grant 9DAAL03-90-G-0093, DOE Grant DEFG05-86-ER25017, and AFOSR Grant 89-0363.

Partially supported by COLCIENCIAS CO: 1106-05-307-93.