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TOPOLOGICAL GROUPS FOR TOPOLOGISTS: PART II

MIKHAIL G. TKACHENKO

Contents

1. Introduction

Here we present the second half of the survey the first part of which appeared in the previuos issue of this journal (see [Tk15]). Primarily, our aim is to systematize the results on free topological groups and show the internal logic of the subject. Therefore, we give no proofs and focus the attention on ideas.

Free topological groups is a flexible and powerful instrument of investigation in the general theory of topological groups. On one side, they serve as a source of numerous examples and on the other, they provide facilities for representing certain topological groups as quotients of topological groups with preassigned properties. In addition, free topological groups is an interesting and quite complicated object of the study which is far from being well understood. This is why we try to give a detailed picture of this somewhat special area.

Our second aim is to discuss briefly several topics in topological groups that have recently been paid much attention by specialists. This includes minimal

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PERIOD MATRIX AND THE TRIGONAL CONSTRUCTION

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Abstract

The main purpose of this paper is to show that the period matrix of a trigonal curve Y depends analytically on the period matrix of the curve X obtained from the trigonal construction and its representation as a $4:1$ cover of $\mathbb{P}^1$

1. Introduction

A curve will always mean a complete nonsingular irreducible curve defined over the complex field $\mathbb{C}$, i.e. a compact Riemann surface. By a $d-$gonal curve of genus g, we mean a pair (C, g_d^1), where C is a curve of genus g together with a base point free linear system g_d^1, or equivalently a morphism $f: C \to \mathbb{P}^1$ of degree d.

The coarse moduli space of $d-$gonal curves of genus g has dimension $\geq g - 2d + 2$, the irreducibility of this space is a well-known fact [EC], R. Donagi and M. Green in [DS], Appendix, p. 9–101, obtained a description of the tangent space of this space at a nice curve C.

Given a tetragonal curve (X, g_4^1) of genus g, one can construct a trigonal curve (Y, g_3^1) of genus $g+1$ together with an unramified double cover $\pi: Z \to Y$. This is known as the trigonal construction and establishes a bijection between the set of tetragonal curves X of genus g and the set of pairs of trigonal curves Y of genus $g+1$ with an unramified double covering $\pi: Z \to Y$. Recillas' theorem states that Jacobians of curves with g_4^1's are the Prym's Jacobians of trigonal curves and conversely (see [Rec2]).

In [HPR] the authors find a relation which describes the Riemann matrix of a trigonal curve of genus $g+1$ in terms of a tetragonal curve of genus g via the trigonal construction.

In this paper we show that there exists a relation between the Teichmüller space of tetragonal curves of genus g and the Teichmüller space of trigonal curves of genus $g+1$. As a consequence of this fact the period matrix of a

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THE LIE ALGEBRA OF THE PURE BRAID GROUP

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Abstract

We study the Lie algebra $E^0_*(P_k)$ associated with the descending central series of the pure braid group P_k, and prove that is a *twisted* extension of free Lie algebras. The *twisting* is given by the infinitesimal braid relations.

1. Introduction

In this article we describe the Lie algebra $E^0_*(P_k)$ associated with the descending central series of the pure braid group P_k. The additive structure of $E^0_*(P_k)$ was computed in [F-R] by M. Falk and R. Randell. The Lie algebra structure was first found by T. Kohno [Ko85] using rational homotopy and mixed Hodge theory, and apart from the grading, it is now known to be isomorphic to the Lie algebra of primitives of $H_*(\Omega F(\mathbb{R}^{2n}, k))$ for $n \geq 2$. In this paper, the structure of the Lie algebra is worked out directly using a presentation of the braid group.

The integral homology of $\Omega F(\mathbb{R}^{2n}, k)$ has been computed by E. Fadell and S. Husseini [F-H] and subsequently by F. Cohen and S. Gitler [C-G], who showed that the Lie algebra of primitives is given by a non-trivial extension of free Lie algebras. Namely, there is an isomorphism from the module of primitives $PH_*(\Omega F(\mathbb{R}^{2n}, q)) \cong L_2 \oplus \cdots \oplus L_q$, where $L_i = L[A_{i,1}, \ldots, A_{i,i-1}]$ is a free Lie algebra on $i-1$ generators of degree $2n-2$, and the relations among the $A_{i,j}$'s are of the form:

$$\begin{aligned} [A_{i,j}, A_{k,i}] &= [A_{k,i}, A_{k,j}] \\ [A_{i,j}, A_{k,j}] &= [A_{k,j}, A_{k,i}] \end{aligned} \qquad \text{for} \quad 1 \leq j < i < k \leq q$$

and $[A_{i,j}, A_{k,l}] = 0$ for distinct i, j, k, l. These are known as the *infinitesimal braid relations* or *Yang-Baxter relations* and they have appeared independently in several contexts in algebra and topology (See for example [Ko87] and [Dri]).

A satisfactory explanation for the connection between both Lie Algebras is still missing, although this appears to be part of a more general phenomenon.

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Keywords and phrases: free Lie algebra, pure braid group, descending central series, configuration spaces.

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WEIGHTED $L^p - L^q$ INEQUALITIES FOR TWO-DIMENSIONAL HARDY OPERATORS WHEN $q < p$

Y. Rakotondratsimba

Abstract

Sufficient condition for the boundedness of the two-dimensional Hardy operator from the weighted Lebesgue space $L^p(]0,\infty[^2, v(y_1,y_2)dy_1dy_2)$ to $L^q(]0,\infty[^2, u(x_1,x_2)dx_1dx_2)$, is obtained whenever $q < p$ with $1 < p < \infty$ and $0 < q < \infty$. A concrete class of weights, for which such a boundedness holds, is also given.

1. Introduction

The two-dimensional Hardy operator is defined as

$$(Hf)(x_1,x_2) = \int_0^{x_1}\int_0^{x_2} f(y_1,y_2)dy_1dy_2, \qquad 0 < x_1, x_2 < \infty.$$

Let $u(\cdot,\cdot) = (u(x_1,x_2))_{x_1,x_2\in]0,\infty[}$ and $v(\cdot,\cdot) = (v(y_1,y_2))_{y_1,y_2\in]0,\infty[}$ be weight functions, e.g. nonnegative measurable functions and locally integrable on $]0,\infty[^2$.

Our purpose in this paper is to derive a sufficient condition on the weights $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$ for which H is bounded from the weighted Lebesgue space $L^p_v = L^p(]0,\infty[^2, v(y_1,y_2)dy_1dy_2)$ to $L^q_u = L^q(]0,\infty[^2, u(x_1,x_2)dx_1dx_2)$ whenever $q < p$ with $1 < p < \infty$ and $0 < q < \infty$. It means thatfor some constant $c > 0$

$$(1.1)\quad \left(\int_0^\infty\int_0^\infty (Hf)^q(x_1,x_2)u(x_1,x_2)dx_1dx_2\right)^{\frac{1}{q}} \leq c\left(\int_0^\infty\int_0^\infty f^p(y_1,y_2)v(y_1,y_2)dy_1dy_2\right)^{\frac{1}{p}}$$

for all functions $f(\cdot,\cdot) \geq 0$. For convenience this boundedness is also denoted by $H : L^p_v \to L^q_u$.

Interest in (1.1) lies on the fact that H plays a fundamental role as an operator controlling many other operators like the Fourier transform, the double

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FILTERING THE COEFFICIENTS OF THE $[2^k]$-SERIES FOR BROWN-PETERSON HOMOLOGY

JESÚS GONZÁLEZ

Abstract

For non-negative integers k and s let $a_{k,s}$ be the s-th coefficient of the universal 2-typical $[2^k]$-series. Thus $a_{k,s}$ is a polynomial over the 2-local integers on variables $v_1, v_2, \ldots$. We study the structure of this polynomial by using a family of filtrations in the coefficient ring of the Brown-Peterson spectrum. As a result we identify k monomials on $a_{k,s}$. The monomials, as well as their 2-divisibility properties, are determined by the dyadic expansion of $s+1$.

1. Introduction

Let BP be the Brown-Peterson spectrum at the prime 2 and let $\mu(X, Y)$ be its formal group law. For a non-negative integer k, the $[2^k]$-series is inductively defined by $[2^{k+1}](T) = \mu([2^k](T), [2^k](T))$, with $[2^0](T) = T$. Thus, the $[2^k]$-series takes the form

$$[2^k](T) = \sum_{s \geq 0} a_{k,s} T^{s+1}, \tag{1.1}$$

where $a_{k,s} \in BP_* = \mathbb{Z}_{(2)}[v_1, v_2, \ldots]$ has dimension $2s$. These algebraic objects play an important role not only in modern algebraic topology (e.g. [3,8]), but also in other branches of mathematics [2,5,10,11]. The coefficients in the $[2^k]$-series seem to carry a lot more geometric information than currently explainable. Although the elements $a_{k,s}$ can be algorithmically computed, they are poorly understood and a knowledge of their global properties is in order. To fulfill in part this task, we generalize ideas in [4] and [7] to take a closer look to the expression of each $a_{k,s}$ as a polynomial in the v_i's. The method is based on the use of a family of filtrations in BP_*. As a consequence we obtain the following

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AREA MINIMIZING SURFACES IN 3-MANIFOLDS

MAX NEUMANN-COTO

Abstract

We look at the immersed incompressible surfaces of smallest area in a closed, orientable, Riemannian 3-manifold and show that in some cases they have minimal intersection among all the immersed incompressible surfaces in the manifold, but that this is far from being true in general.

Let M be a closed, orientable 3-manifold, with some Riemannian metric. A closed orientable surface F immersed in M is called *incompressible* if it is not a sphere and the induced map $\pi_1(F) \to \pi_1(M)$ is injective. F is a *least area surface* if it minimizes area among all the surfaces freely homotopic to it.

When M is irreducible (i.e., every sphere embedded in M bounds a 3-ball in M) then every immersed incompressible surface in M is freely homotopic to a smoothly immersed least area surface ([7] [9]). In [2] Freedman, Hass and Scott showed that the least area surfaces in an irreducible M *intersect least*, i.e., they have the minimum number of intersections and self-intersections allowed by their free homotopy classes, if the intersections are counted carefully. So, for example, a least area surface which is freely homotopic to an embedded surface is either embedded or double-covers a nonorientable embedded surface.

One can ask if the surfaces that minimize area among *all* immersed incompressible surfaces in the manifold (let's call them *absolutely least area surfaces*, or ALAS) also have minimal intersections among all those surfaces, and if something similar happens with the collections of immersed incompressible surfaces that minimize area in a homology class in $H_2(M, Z)$. This does not follow from [2], because in general there are no relations between the minimum areas in different homotopy classes of surfaces, except when one is homotopic to a covering of the other (then a least area surface in the first class must cover a least area surface in the second class, and so the areas are related by the degree of the covering).

For the manifolds fibred by circles (Seifert fibred manifolds) we find some inequalities between the minimum areas in different homotopy classes of surfaces that hold for all Riemannian metrics. These inequalities imply that in the product manifolds $F \times S^1$ the ALAS are always embedded, and that the

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AN INEQUALITY BETWEEN ENTROPIES OF PSEUDOGROUPS OF HOLOMORPHIC GERMS

XAVIER GÓMEZ-MONT AND BRUNO WIRTZ

Abstract

For Axiom A diffeomorphisms on compact manifolds, the entropy of fixed points coincides with the topological entropy (see Mañe [6] and Katok and Hasselblatt [5]). Using Ghys, Langevin and Walczak [3], one extends the definitions of entropies to pseudogroups of germs of biholomorphisms $\mathrm{Bih}_{\mathbb{C},0}$. The topological entropy of these pseudogroups has been estimated in [7] and there exists fixed points for these pseudogroups [4]. In this paper we show that the entropy of fixed points is greater than or equal to the topological entropy for generic pseudogroups of germs of biholomorphisms $\mathrm{Bih}_{\mathbb{C},0}$ restricted to small annuli around 0.

0. Introduction

We present a construction that produces fixed points for pseudogroups $\mathcal{P}$ of holomorphic maps on an annulus. The pseudogroups $\mathcal{P}$ we consider are generated by a finite number of germs $H_1 = \{g_1^{\pm 1}, \ldots, g_r^{\pm 1}\} \subset \mathrm{Bih}_{\mathbb{C},0}$ of biholomorphisms of the complex plane $(\mathbb{C}, 0)$ restricted to a small annulus $\mathbf{A}_\beta^\alpha := \{\beta \leq |z| \leq \alpha\}$. Denote the maximal expansivity of the generators by

$$(0.1)\qquad \begin{aligned} M &:= \max\{\, |(g_j^{\pm 1})'(z)| \ / \ z \in \bar{\Delta}(0,\alpha),\ j = 1, \ldots, k\}, \\ \Delta(z_0, s) &:= \{z \in \mathbb{C} \ / \ |z - z_0| < s\}. \end{aligned}$$

The words of the pseudogroup $\mathcal{P}$ of length n are obtained by a finite juxtaposition of n elements of H_1, and the domain of definition of the word $f := g_{i_n} \circ \cdots \circ g_{i_1}$ is the open set in $\mathbf{A}_\beta^\alpha$ where all the partial words $g_{i_\ell} \circ \cdots \circ g_{i_1}$ of f are defined and take values in the annulus $\mathbf{A}_\beta^\alpha$, for $\ell = 1, \ldots, n$.

We say that x and y are strictly (n, ε) separated if for all words h of the pseudogroup of length less than n and which are defined for x and for y we have that $\mathrm{dist}(h(x), h(y)) < \varepsilon$, but there is a word f in the pseudogroup of

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ESTIMATION OF THE REGION OF ATTRACTION OF THE ORIGIN OF CLOSED-LOOP LINEAR SYSTEMS WITH SATURATED LINEAR CONTROLLERS: A FIRST HARMONIC APPROACH

BALTAZAR AGUIRRE, JOSÉ ALVAREZ-RAMÍREZ AND RODOLFO SUÁREZ

Abstract

The aim of this paper is to study the existence of first harmonic periodic orbits for linear systems with saturated linear controllers. In particular, bifurcations on the number of first harmonic periodic orbits are analized. For open-loop antistable systems a Hopf bifurcation occurs, implying that two of the first harmonic periodic orbits correspond to periodic orbits. These results are used to guide and facilitate simulation procedures to estimate the region of attraction of the origin.

1. Introduction

In this paper the following system is considered

$$\dot{y}(t) = Ay(t) + b\xi(t) \tag{1.1}$$

where $y \in \mathbb{R}^3, u \in \mathbb{R}$, $A \in \mathcal{M}_{3\times 3}$, $b \in \mathbb{R}^3$, (A, b) is a controllable pair (see [3] and [19]) and the state feedback $\xi = \text{sat}(u)$ is defined by the saturation function

$$\text{sat}(u) = \begin{cases} -1 & \text{if } u \leq -1 \\ u & \text{if } -1 < u < 1 \\ 1 & \text{if } 1 \leq u \end{cases}$$

with $u(t) = k^T y(t)$ chosen such that $A + bk^T$ is a *Hurwitz* matrix, that means $\sigma(A + bk^T) \subset \mathbb{C}^-$, where $\sigma(A + bk^T)$ denotes the spectrum of the matrix $A + bk^T$. This implies that the origin is a locally asymptotically stable equilibrium point of (1.1). Our main problem is to study the periodic orbits induced by the saturated stabilizing linear feedback ξ. Although the approach we use is of approximating nature, the obtained results allow us to discuss some facts about the region of attraction of the origin, denoted $\Omega(0)$.

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