Control of Composite Distributed Parameter Systems *

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Abstract

In this paper we consider higher order numerical methods for approximating composite control systems defined by coupled hyperbolic PDEs and ordinary differential equations. The work is motivated by applications to optimization and control of thermal management systems and systems with delays. We begin with a simple model of a counterflow heat exchanger found in [2], [3] and [4], where one includes the effect of axial conduction and boundary control inputs with actuator dynamics. This "full-flux" model is described by the coupled system

$$\begin{array}{l} \frac{\partial T_1(t,x)}{\partial t} = \mu_1 \frac{\partial^2 T_1(t,x)}{\partial x^2} - v_1(t,x)\frac{\partial T_1(t,x)}{\partial x} + h_1\left[T_2(t,x) - T_1(t,x)\right] \\ \frac{\partial T_2(t,x)}{\partial t} = \mu_2 \frac{\partial^2 T_2(t,x)}{\partial x^2} + v_2(t,x)\frac{\partial T_2(t,x)}{\partial x} + h_2\left[T_2(t,x) - T_2(t,x)\right] \end{array}, \quad (1)$$

where the constants h_1, h_2 are heat transfer coefficients, μ_1, μ_2 are diffusion coefficients and $v_1(t,x)$, $v_2(t,x)$ are flow velocities for channels one and two, respectively. The flow velocities $v_1(t,x)$ and $v_2(t,x)$ are possible control inputs.

For channel one we have the boundary conditions

$$T_1(t,0) = v(t), \quad \mu_1[T_1]_x(t,L) = 0, \quad (2)$$

where $v(\cdot)$ is a "boundary control term" and for channel two we have

$$-\mu_2[T_2]_x(t,0) = 0, \quad T_2(t,L) = 0. \quad (3)$$

Initial conditions for each channel are given by

$$T_1(0,x) = \varphi(x) \ \text{ and } \ T_2(0,x) = \psi(x), \quad 0 < x < L. \quad (4)$$

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In addition, we assume the actuator dynamics are described by a finite dimensional system of the form

$$\dot{\mathbf{w}}_a(t) = \mathbf{A}_a\mathbf{w}_a(t) + \mathbf{B}_a\mathbf{u}(t), \tag{5}$$

or a delay differential equation

$$\dot{\mathbf{w}}_a(t) = \mathbf{A}_0\mathbf{w}_a(t) + \mathbf{A}_1\mathbf{w}_a(t-r) + \mathbf{B}_a\mathbf{u}(t), \tag{6}$$

with output

$$v(t) = \mathbf{H}_a\mathbf{w}_a(t), \tag{7}$$

where $\mathbf{A}_a, \mathbf{A}_0$ and $\mathbf{A}_1$ are $n \times n$ matrices, $\mathbf{B}_a$ is an $n \times m$ matrix and $r > 0$ is a delay.

We formulate these systems as abstract composite distributed parameter systems of the form

$$\begin{aligned} \dot{\boldsymbol{z}}_1(t) &= \boldsymbol{\mu}\mathcal{A}_1\boldsymbol{z}_1(t) + \boldsymbol{v}(t)\mathcal{H}_1\boldsymbol{z}_1(t) + \mathbf{H}_1\boldsymbol{z}_1(t) + F\boldsymbol{z}_2(t)(t) + \mathbf{B}_1\mathbf{u}(t) \\ \dot{\boldsymbol{z}}_2(t) &= \mathcal{A}_2\boldsymbol{z}_2(t) + \mathbf{B}_2\mathbf{u}(t). \end{aligned} \tag{8}$$

If in addition, $v_1(t,x)$ and $v_2(t,x)$ are control inputs, then (8) becomes an abstract bilinear control system of the type found in [1]. We use this framework to discuss discuss well-posedness and computational methods for approximation. In particular and compare finite element (FE), finite volume (FV) and combined FE-FV methods for optimization and control of such systems. These schemes are applied to a simple numerical example to illustrate the idea.

References

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