Irena Lasiecka
and Katarzyna Szulc, Antoni Zochowski.
Department of Mathematical Sciences, University of Memphis
and IBS-Polish Academy of Sciences, Warsaw.

Title: Flow Control and Drag Reduction in a 3-D Channel by Boundary Actuators.

Abstract Fluid structure interaction comprising of an elastic body immersed in the moving fluid is considered. The fluid is modeled by an incompressible Navier-Stokes equations while the structure is modeled by a 3-D system of elasticity. The interaction between the structure and the fluid takes place on an interface between the two environments. Since the body is moving within the fluid-with the fluid's velocity denoted by w, the interface is defined by a free [unknown] surface which depends on the displacement of the body denoted by $\mathbf{u}$. Thus it becames, by itself, an unknown variable $\Gamma(u)$ for the given problem. The resulting system, after moving to Lagrangian coordinates, becomes a quasilinear PDE. For such model, our aim is to construct an optimal control which would reduce a drag exercsized by the fluid on the obstacle. There are several choices for the cost objective to select from. We opt for a minimization of hydrostatic pressure which can be represented by the shape functional :

$$J(\mathbf{g}) \equiv \int_{\mathbf{\Gamma(u)}} \mathbf{p(w(g), u)n \cdot e_1 ds} \tag{1}$$

where $p(\mathbf{w}, \mathbf{u})$ denotes the pressure determined by the Navier Stokes equation forced by the control $\underset{\sim}{g}$ and evaluated at the interface between the solid and the fluid. $\mathbf{n}$ denotes a normal direction to the boundary $\Gamma(\mathbf{u})$. The control function $\mathbf{g}$ -fluid's intake at the inlet, is subject to volume constraints. The goal is to reduce a drag of the obstacle by changing the flow profile on the inlet. This leads to a boundary control problem with a minimization of a hydro-elastic pressure on the interface between the solid and the fluid. The latter is expressed by the form of shape functional given in (1). The interface $\Gamma(\mathbf{u})$ is "free" and it is by itself an unknown variable. The problem is reformulated as a quasilinear PDE-control with a free boundary. It is well recognized that mathematical challenge of the problem under study is due to (i) quasilinear nature of the state equations, (ii) the lack of coercivity in the functional cost [1]. Moreover, the additional challenge is introduced by boundary conditions imposed on the external domain of the fluid. While the boundary conditions imposed on the interface are physical and correspond to matching velocities and matching stresses, the boundary conditions imposed on the external boundary are typically Dirichlet-Neumann type imposed on the fluid. More specifically, the boundary conditions on the outlet are given by the Neumann free data, while boundary conditions on the inlet and lateral walls are Dirichlet. Thus we deal with the so called "mixed" boundary conditions imposed on the fluid domain [change from Dirichlet to Neuman]. This feature is well known to cause singularities in elliptic solutions. Handling of these requires a careful analysis of local singularities which depend on the geometry of the domain. On the other hand, quasilinear nature of the state equation requires working with sufficiently smooth solutions, while Dirichlet-Neumann BC prevent sufficient smoothness and lead to a formation of singularities. Thus, the challenge in solving the problem is to strike

a right balance between the regularity of solutions needed to handle the nonlinear nature of the problem and the regularity conforming with "mixed" boundary conditions. This requires geometric and microlocal localization of the problem with separate treatments near the interface and near the external part of the boundary. The commutators are critical in handling of the coupling.

The first result discussed in this talk provides an existence of optimal control [with volume constraints] which minimizes the drag of the obstacle, under the assumption of sufficiently small strains and time independent dynamics [2]. This result relies critically on the regularity and differentiability of the control to state map -the results obtained in [5, 1] which are very first steps in the analysis of control problem. As for the optimization- the difficulties caused by the lack of convexity and the compromised regularity of solutions due to singularities caused by mixed boundary conditions - are handled by a suitable use of compensated compactness methods . The latter allow to establish a variant of weak lower-semicontinuity for the objective cost. The obtained results are illustrated by numerical simulations which confirm and provide an interpretation of the theoretical findings. Numerical simulations with shape functional provide evidence for effectiveness of the optimization algorithm proposed.

In the second part of the talk we shall discuss time dependent evolutions with Dirichlet control on the inlet and Dirichlet data on the outlet. Here, we shall rely on recent results on maximal regularity for Stoke's operator along with hidden trace regularity for elastodynamic system. Suitable construction of global [in time] solution lies in the heart of the matter [4]. [Loal solutions have been constructed earlier in [3]]. The obtained result provides an existence of optimal control under the assumption of small strains. The non-autonomous model with "mixed" [Dirichlet-Neumann] boundary data is under further studies.

References

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